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# PARAMETRIZATION OF OPEN-OCEAN DEEP CONVECTION

## PART 1: THEORETICAL DESCRIPTION OF A PLUME ENSEMBLE MODEL

by

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May 1993

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# PARAMETRIZATION OF OPEN-OCEAN DEEP CONVECTION

# PART I: THEORETICAL DESCRIPTION OF A PLUME ENSEMBLE MODEL

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## 1. INTRODUCTION

Convection takes place throughout the worlds oceans. There are two distinct types: open-ocean and boundary (or slope ) convection. Open-ocean convection takes place away from the influence of the ocean shelf and floor. On the other hand, boundary or slope convection is associated with convection down or along bottom slopes (see Killworth, 1983, for a review). Only open-ocean convection will be considered here.

Open-ocean convection occurs as intense turbulent plumes which take water from a relatively shallow surface layer to the deep ocean. These plumes are triggered when the surface layer becomes denser than the water immediately below. This occurs when there is intense surface cooling and/or evaporation. When ice is forming the density of the surface layer can also increase through the release of salt (brine rejection). In order that the triggered plumes can penetrate into the deep ocean the vertical static stability must be low. Deep convection is therefore associated with areas which are "preconditioned" to having a low vertical static stability as well as strong wintertime surface forcing. The main areas are therefore found in the high to polar latitudes. Areas where convection has been observed include: the Labrador Sea, Greenland Sea, Bransfield Strait, Weddell Sea, regions of the sub-polar oceans and the Mediterranean Sea (where the Mistral provides strong winter cooling in the Gulf of Lions)

The plumes, characteristic of deep convection, have horizontal scales of order of a few kilometres or less. At present, Ocean General Circulation Models (OGCMs) used for basin scale studies have horizontal length scales which are tens to hundreds of kilometres. These clearly cannot resolve the convective plumes, yet the plumes transport large amounts of water from the surface to the deep ocean. Most models crudely attempt to represent the effects of deep convection by the process of convective adjustment or a large vertical diffusivity. "Convective adjustment" is essentially the mixing of adjacent layers in the vertical if the top one is more dense than the one below. Variations on this basic idea lead to some schemes which partially remove the convective instability while others fully remove the instability and homogenise the water column. In reality adjacent layers of the ocean do not mix. Instead, water from the unstable surface layer sinks into the deep ocean and is deposited near the depth where its buoyancy becomes neutral. During the sinking process there is some mixing with the surrounding.

A more realistic way of representing deep convection as a sub-grid scale process in ocean models is required. This paper develops a theory for a plume model to tackle that problem. In section two a model is developed to represent an ensemble of convective plumes within an area, say a grid column of an ocean general circulation model. Section three describes the starting or boundary conditions for the plume model. The model and its boundary conditions contain various parameters whose value is not certain. These are discussed in section 4. The last section provides a summary.

## 2. PLUME ENSEMBLE MODEL

Basic equations for a plume driven by buoyancy and modified by mixing with its environment, are derived in this section. The plume model is then generalised to incorporate the characteristics of an ensemble of plumes.

### Plume Equations

*Conservation equation*

A plume is driven by gravitational acceleration because it is denser than the surrounding water , its "environment". As the plume sinks water from its surroundings is entrained into the plume by turbulent mixing at the plume's edges. Similarly some of the plume water is detrained from the plume and mixed with the environment .

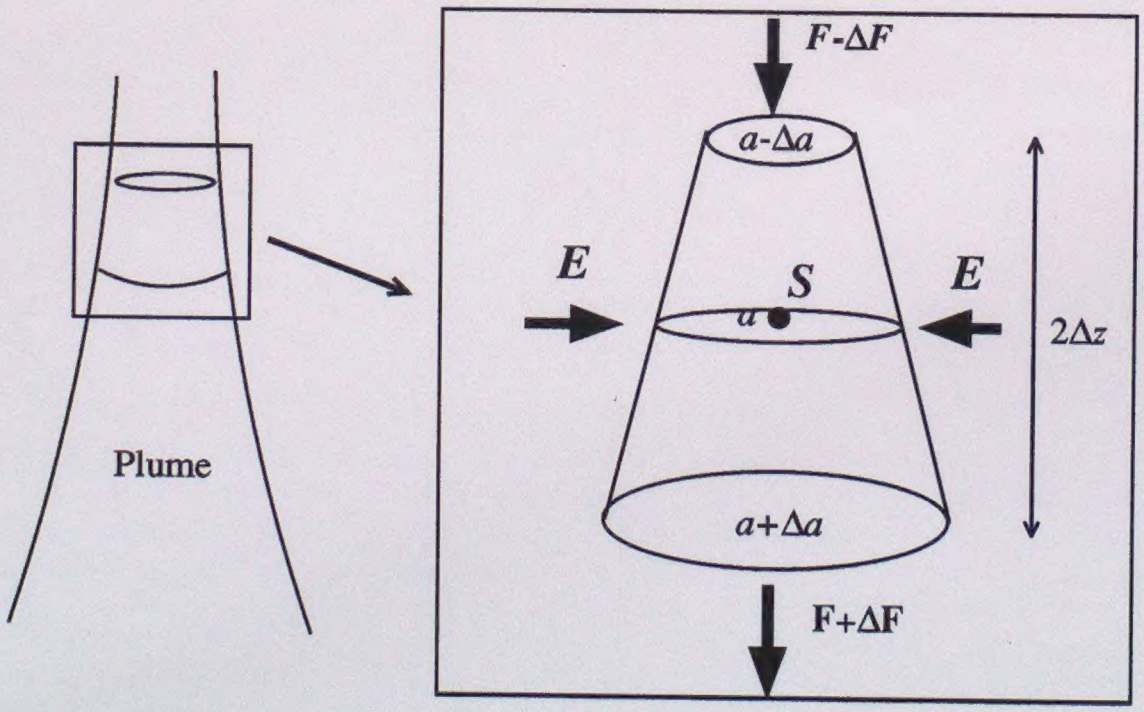


**Figure 1.** Plume element

Consider some element of the plume of height $2\Delta z$ (see figure 1) whose midpoint is at depth

$z$, where the cross sectional area of the plume is $a(z)$. It will be assumed that the plume has a horizontal top hat profile, that is, that all variables describing the plume have no horizontal variation, only depth dependence. As in fig. 1

$$a(z-\Delta z)\approx a-\Delta a$$

$$a(z+\Delta z)\approx a+\Delta a$$

Assuming the element to be locally conical, its volume is $V\approx 2a\Delta z$ and the area of the "quasi-vertical" sides of the element is $A_V=2c\Delta z$ where c is the circumference of a horizontal cross section of the element at height $z$. Suppose that there is some flux $F_X(z)$ per unit area of some quantity $X$ passing through the element. Further suppose that there is some amount $E_X$ of the same quantity $X$ per unit area per unit time being entrained (or detrained) through the "quasi-vertical" sides of the plume element and that there is some source $S_X$ of $X$ per unit volume per unit time within the element. Assuming that there is no other source of the quantity $X$, conservation of that quantity within the plume element can be written as

$$(F_X+\Delta F_X)(a+\Delta a)-(F_X-\Delta F_X)(a-\Delta a)=A_V E_X+VS_X$$

which on simplifying gives

$$2(\Delta F_X a+F_X\Delta a)=A_V E_X+VS_X$$

Substituting for $A_V$ and $V$ and taking the limit as $\Delta z\rightarrow 0$ gives

$$a\frac{dF_X}{dz}+F_X\frac{da}{dz}=cE_X+aS_X$$

which can be written as

$$\frac{d}{dz}(F_X a)=cE_X+aS_X \qquad (2.1)$$

This is the conservation equation for some quantity $X$ with vertical flux $F_X$, volume source $S_X$ and entrainment (detrainment) $E_X$ along its sides.

*Conservation of Mass*

Equation (2.1) can be applied to mass, where $F_X$ becomes $F_M$ the vertical mass flux per unit cross-sectional area of the plume. $F_M$ can be written as

$$F_M=\rho_p w$$

where $\rho_p(z)$ is the density of the plume and $w(z)$ is its downward vertical velocity. Representing the mixing through the sides by water moving from the environment into the plume with some entrainment velocity $v_E(z)$ and from the plume to the environment with detrainment velocity $v_D(z)$, $E_M$ can be written as

$$E_M=\rho_e v_E - \rho_p v_D \qquad (2.2)$$

Following other authors it is assumed that the entrainment velocity is some fraction $\varepsilon$ of the vertical velocity (eg. Turner, 1973). Similarly it is assumed that the detrainment velocity is some fraction $\delta$ of the vertical velocity $w$ (this is done in atmospheric convection parametrization, see for example Gregory and Rowntree, 1990). Thus

$$E_M=\rho_e \varepsilon w - \rho_p \delta w$$

Since mass cannot be generated within the element of the plume, $S_M=0$. Equation (3.1) then leads to the equation for mass conservation

$$\frac{d}{dz}(\rho p w a) = wc(\rho_e \epsilon - \rho p \delta)$$

where $wc\rho_e\varepsilon$ represents the entrainment of mass from the environment into the plume and $wc\rho_p\delta$ the detrainment of mass from the plume to the environment.

*Momentum Conservation*

Equation (2.1) can also be applied to the conservation of vertical momentum. The vertical flux of momentum per unit area is given by $F_M=\rho_p w^2$. Water being entrained into the plume will have the momentum of the environment and water detrained out of the plume will have the momentum of the plume. The momentum entrainment flux per unit area is therefore $\varepsilon\rho_e w_e w$, where $w_e$ is the vertical velocity within the environment. Similarly the momentum detrainment flux per unit area is $\delta\rho_p w^2$. Therefore

$$E_w=\varepsilon\rho_e w_e w-\delta\rho_p w^2$$

If the contribution to the vertical velocity, $w_e$, from the large scale flow is ignored so that $w_e$ becomes the local perturbation to the convective plume, then by conservation of mass

$$(A-a)w_e=-aw$$

where $A$ is the total horizontal cross-sectional area of the plume and its environment.

Putting $\alpha=a/A$, the fractional area occupied by the plume, gives

$$w_e = -\frac{\alpha}{(1-\alpha)}w$$

and so

$$E_w = - w^2\left(\frac{\alpha}{(1-\alpha)}\epsilon\rho_e + \rho p \delta\right)$$

Since, in general, the plume has a different density from its environment, this density difference accelerates the water particles within the plume and provides a source of momentum. The buoyancy source of momentum per unit volume is given by

$$S_w=\rho_p g'$$

where $g'$ is the reduced gravity given by

$$g' = \frac{(\rho_p-\rho_e)}{\rho_0} g \qquad (2.3)$$

where $g$ is the acceleration of free fall and $\rho_0$ is a reference density.

Using $F_W$, $E_W$ and $S_W$ in equation 2.1 produces the equation for the conservation of plume momentum

$$\frac{d}{dz}(\rho_p a w^2) = -w^2 c\left(\frac{\alpha}{(1-\alpha)}\epsilon\rho_e + \delta\rho_p\right) + a\rho_p g'$$

*Conservation of Heat*

For an adiabatic system the conservation of heat corresponds to the conservation of potential temperature, this being conserved as a particle within the plume sinks adiabatically. If $F_\theta(z)$ is the vertical flux of thermodynamic energy or heat then

$$F_\theta=\rho_p c_p \theta_p w$$

where $\theta_p(z)$ is the potential temperature of the plume referenced to the surface and $c_p$ is the specific heat capacity of sea water (at the surface).

The water being entrained into the plume will, in general, have a different temperature to that of the plume. The heat flux due to this entrainment is $\varepsilon w c_p \rho_e \theta_e$ where $\theta_e$ is the potential temperature of the environment. Similarly detrainment from the plume will result in a heat loss whose flux is $\delta w c_p \rho_p \theta_p$. The net heat flow per unit area into the plume due to entrainment and detrainment is therefore

$$E_q=w c_p(\varepsilon\rho_e\theta_e-\delta\rho_p\theta_p) \qquad (2.4)$$

Since the vertical motion within the plume is assumed to be adiabatic, $S_\theta=0$.

Using $S_\theta$, $E_\theta$, $F_\theta$ in equation 2.1 gives for the conservation of heat (or potential temperature)

$$\frac{d}{dz}(\rho_p\theta_p a w) = cw(\epsilon\rho_e\theta_e - \delta\rho_p\theta_p) \qquad (2.5)$$

*Salt Conservation*

In a similar manner that heat is conserved, salt is also conserved. The vertical salt flux $F_S$ is given by $F_S=\rho_p S_p a w$ where $S_p(z)$ is the salinity of the plume. The entraining water will represent an entraining salt flux of $\varepsilon w \rho_e S_e$ where $S_e$ is the salinity of the environment and the detraining water will have an associated detraining salt flux per unit area of $\delta w \rho_p S_p$. The net entrainment/detrainment salt flux per unit area is therefore

$$E_S = w(\rho_e \varepsilon S_e - \rho_p \delta S_p)$$

There can be no source of salt within the plume element and so $S_S = 0$. Again using 2.1 with $F_S$, $E_S$ and $S_S$ gives the equation for salt conservation

$$\frac{d}{dz}(\rho_p S_p a w) = cw(\epsilon \rho_e S_e - \delta \rho_p S_p)$$

## Mass Flux Equations

So far the plume is described by the four variables potential temperature $\theta_p$, salinity $S_p$, vertical velocity $w$ and horizontal cross-sectional area $a$. These are related by equations 2.2, 2.3, 2.4, 2.5 and $\rho = \rho(\theta, S, z)$. The cross sectional area $a$ can be replaced by defining a new variable $M_p$, the total mass flux for the plume given by

$$M_p = \rho_p w a$$

Equations 2.2 to 2.5 then become

$$\frac{d}{dz} M_p = wc\left(\rho_e \epsilon - \rho_p \delta\right)$$

$$\frac{d}{dz}(M_p w) = -w^2 c\left(\epsilon \frac{\alpha}{(1-\alpha)} \rho_e + \delta \rho_p\right) + \frac{M_p}{w} g'$$

$$\frac{d}{dz}(M_p \theta_p) = cw(\epsilon \rho_e \theta_e - \delta \rho_p \theta_p)$$

$$\frac{d}{dz}(M_p S_p) = cw(\epsilon \rho_e S_e - \delta \rho_p S_p)$$

Expanding these equations and using the mass flux equation to eliminate $dM_p/dz$ from the others gives

$$\begin{aligned}
\frac{dM_p}{dz} &= wc(\rho_e \epsilon - \rho_p \delta) \\
\frac{dw}{dz} &= -\frac{w^2 c \epsilon \rho_e}{M_p(1-\alpha)} + \frac{g'}{w} \\
\frac{d\theta_p}{dz} &= -\frac{wc}{M_p} \epsilon \rho_e (\theta_p - \theta_e) \\
\frac{dS_p}{dz} &= -\frac{wc}{M_p} \epsilon \rho_e (S_p - S_e)
\end{aligned} \qquad (2.6)$$

These equations describe the plume's mass flux $M_p$, vertical velocity $w$, potential temperature $\theta_p$ and salinity $S_p$. Note that $\alpha$ is the fractional area occupied by the plume, $a/A$. In terms of $M_p$ it is given by

$$\alpha = \frac{M_p}{\rho_p w A}$$

## Plume equations with ensemble characteristics

Given some area of the ocean, say $A$, where convection is taking place, there will be some number of plumes $n$ (an ensemble). Area $A$ could, for example, represent the horizontal area of a grid box in an ocean general circulation model. With the view of producing a scheme which represents the collective characteristics of such an ensemble of plumes, define $M$ to be the total mass flux of the $n$ plumes

$$M = \sum^{n} M_p$$

If we now assume that all members of the ensemble are identical with vertical velocity $w$, potential temperature $\theta_p$ and salinity $S_p$, then equations 2.6 can be modified to represent the ensemble mass $M$, potential temperature $\theta_p$, salinity $S_p$ and vertical velocity $w$, and are written as

$$\begin{aligned}
\frac{dM}{dz} &= wC(\rho_e\epsilon - \rho_p\delta) \\
\frac{dw}{dz} &= -\frac{w^2C\epsilon\rho_e}{M(1-\alpha)} + \frac{g'}{w} \\
\frac{d\theta_p}{dz} &= -\frac{wC}{M}\epsilon\rho_e(\theta_p - \theta_e) \\
\frac{dS_p}{dz} &= -\frac{wC}{M}\epsilon\rho_e(S_p - S_e)
\end{aligned}$$

where $C$ is the total circumference of all the plumes in the ensemble. Thus

$$C = \sum^{n} c$$

For a given horizontal shape of the plumes the circumference can be written as $c=\lambda a_p^{1/2}$ ($\lambda$ is a constant for a given shape. For an ensemble $C=\Lambda a^{1/2}$ where $C$ is the total horizontal circumference and $a$ the total horizontal area of the ensemble ($\Lambda$ is a constant for a given plume shape, see section 4). Using

$$a = \frac{M}{\rho_p w}$$

the plume ensemble equations become

$$\begin{aligned}
\frac{dM}{dz} &= \Lambda M\left(\frac{w}{\rho_p M}\right)^{\frac{1}{2}}(\rho_e\epsilon - \rho_p\delta) \\
\frac{dw}{dz} &= -\Lambda w\left(\frac{w}{\rho_p M}\right)^{\frac{1}{2}}\frac{\epsilon\rho_e}{(1-\alpha)} + \frac{g'}{w} \\
\frac{d\theta_p}{dz} &= -\Lambda\left(\frac{w}{\rho_p M}\right)^{\frac{1}{2}}\epsilon\rho_e(\theta_p - \theta_e) \\
\frac{dS_p}{dz} &= \Lambda\left(\frac{w}{\rho_p M}\right)^{\frac{1}{2}}\epsilon\rho_e(S_p - S_e)
\end{aligned}$$

It is now assumed that density variations are only important in the g' term; this is similar to

the Boussinesq assumption. Replacing each density in the above equations by a reference density $\rho_0$, except in the g' term gives the final plume equations

$$\frac{dM}{dz} = \Lambda M\left(\frac{w\rho_0}{M}\right)^{\frac{1}{2}}(\epsilon - \delta)$$

$$\frac{dw}{dz} = -\Lambda w\left(\frac{w\rho_0}{M}\right)^{\frac{1}{2}}\frac{\epsilon}{(1-\alpha)} + \frac{g'}{w}$$

$$\frac{d\theta_p}{dz} = -\Lambda\left(\frac{w\rho_0}{M}\right)^{\frac{1}{2}}\epsilon(\theta_p - \theta_e)$$

$$\frac{dS_p}{dz} = -\Lambda\left(\frac{w\rho_0}{M}\right)^{\frac{1}{2}}\epsilon(S_p - S_e)$$

These are the final set of equations which will be used to describe the characteristics of the plume ensemble. They can be solved numerically given the environmental potential temperature $\theta_e(z)$ and salinity $S_e(z)$ and that the boundary (or starting) conditions $M_0, w_0, \theta_{p0}$ and $S_{p0}$ at depth $z_0$, the depth where the plume starts, are known. The boundary (or starting) conditions will be described in the next section.

## 3. BOUNDARY (STARTING) CONDITIONS

*Potential temperature and salinity*

When the surface of the ocean is cooled a gradient in potential temperature develops such that potential temperature increases with depth within a surface layer. Similarly, when there is evaporation a salinity gradient develops with salinity decreasing with depth. Both these processes lead eventually to the setting up of a density gradient which is conditionally unstable (see later), density decreasing with depth (see for example, Anis and Moun, 1992 or Leaman and Schott, 1991). Other processes such as brine release during ice formation, advection etc. (see Chu, 1991) may also lead to the setting up of a vertical density gradient. When the vertical density gradient becomes unstable convective plumes develop. These act in such a way that they remove the heavier water from the density inversion and upwell less dense water, thus reducing the unstable density gradient.

Suppose that the depth of the surface density inversion is $z_0$. This depth is determined by the diffusive and other mixing processes taking place within the ocean, including the convective plumes themselves. The density inversion only becomes unstable when a critical value is reached for the Rayleigh number associated with it. The Rayleigh number can be defined as

$$Ra = g\left(\frac{\Delta\rho}{\rho_0}\right)\frac{z_0^3}{K_\rho K_\nu}$$

where $\Delta\rho$ is the density difference between the top and bottom of the inversion, ie $\Delta\rho=\rho(z=0)-\rho(z=z_0)$, $\rho_0$ is a reference density, $K_\rho$ is an eddy diffusion coefficient for density and $K_v$ is the eddy viscosity. Convection takes place when $Ra \geq Ra_c$, $Ra_c$ being the critical value of the Rayleigh number.

It is now assumed that the plumes start at depth $z_0$ and take their mass from the inversion layer. The temperature and salinity of the plume water will be that of the inversion layer. Since within the inversion layer the potential temperature and salinity vary with depth, their mean values will be used to represent the plume so that

$$\theta_p = \frac{1}{z_0}\int_0^{z_0} \theta_e(z)dz$$

$$S_p = \frac{1}{z_0}\int_0^{z_0} S_e(z)dz$$

where $\theta_p$ and $S_p$ are the starting plume potential temperature and salinity at $z_0$ respectively and $\theta_e(z)$ and $S_e(z)$ are the environmental potential temperature and salinity respectively (known a priori, e.g. from previous OGCM timestep).

*Vertical velocity and mass flux*

It now remains to deduce the plume's vertical velocity $w_0$ and mass flux $M_0$ at its starting point. Suppose that in some horizontal area $A$ of the ocean, the plumes within that area occupy an area $a$, defined at the staring point of the plumes. Let $\alpha=a/A$ be the fractional area of the plumes compared to the whole area in which the plumes exist. Suppose further that at the starting point (depth $z_0$) the vertical velocity in the plume is $w_0$ and in the environment $w_e$ (upwards). Conservation of mass in the upper layer leads to

$$(1-\alpha)w_e=\alpha w_0 \qquad (3.1)$$

The upper layer is denser than that below it and this results in the generation of momentum within the upper layer at a rate $Az_0g'$ where $g'=\Delta\rho/\rho_0$ is the reduced gravity and $\Delta\rho=\rho_p-\rho_e$ is the difference in density between the plume $\rho_p(\theta_p,S_p)$ and the environment $\rho_e(\theta_e,S_e)$ at $z_0$.

Vertical momentum in the upper layer is conserved by removing water from the layer into the plume, thus removing vertical momentum at a rate $aw_0^2$. The vertical momentum generated in the upper layer is also partly neutralised by mixing with water of negative vertical momentum resulting from up welling in the environment. This represents an upward vertical momentum flux of $(A-a)w_e^2$. Thus

$$Az_0g'=aw_02+(A-a)w_e^2$$

which in terms of $\alpha$ gives

$$z_0g'=\alpha w_0^2+(1-\alpha)w_e^2 \qquad (3.2)$$

Eliminating $w_e$ from equations (3.1) and (3.2) leads to

$$w_0 = \left(\frac{z_0 g'}{\alpha}\right)^{\frac{1}{2}} \qquad (3.3)$$

Thus the vertical velocity can only be known if $\alpha$ is known.

The initial mass flux is given by $M_0=\rho_0 w_0 a$ and after substituting for $w_0$ and $a$ in terms of $\alpha$ gives

$$M_0=(\alpha z_0 g')^{1/2}\rho A$$

In order to deduce a value for $\alpha$ the plume is projected to the surface from its starting point $z_0$. It is assumed that the plume starts at the surface with zero vertical velocity and is driven by buoyancy given by $g'=g\Delta\rho/\rho_0$. As $\Delta\rho$ is the average for the layer $z_0$ it already represents the effects of entrainment and mixing between the plume and the environment within the layer $z_0$. The vertical velocity is therefore only dependent on $g'$ and is described by

$$\frac{dw}{dz} = \frac{g'}{w}$$

with the solution for $w$ at $z_0$ being $w_0=(2g'z_0)^{1/2}$. Substituting this into (3.3) leads to $\alpha=0.5$ and

$$M_0 = \left(\frac{z_0 g'}{2}\right)^{\frac{1}{2}} \rho A$$

Thus the plumes (or plume) occupy half of the area in which they have influence at their staring point as defined here.

## 4. PLUME MODEL PARAMETERS

### Plume Model Free Parameters

*Entrainment Coefficient* $\varepsilon$

The entrainment coefficient $\varepsilon$ is a measure of turbulent mixing at the plumes' edges. The entrainment velocity of the fluid particles is expressed as a fraction $\varepsilon$ of the plume vertical velocity. Turner (1973) used $\varepsilon=0.1$ and this is now widely used. $\varepsilon$ is here initially taken to be a constant and equal to 0.1.

*Detrainment Coefficient* $\delta$

Detrainment represents the turbulent mixing which mixes plume water with environment water. The detrainment coefficient $\delta$ is a measure of the intensity of this mixing. The detraining parcels are assumed to have a velocity which is a fraction $\delta$ of the

vertical velocity. There is little in oceanographic literature on detrainment. In atmospheric convective parametrizations $\delta$ is assumed to be smaller than the entrainment coefficient $\varepsilon$ while the plume remains positively buoyant, for example Gregory and Rowntree, 1990, uses $\delta \sim 1/3\varepsilon$. Once the plume goes past its neutral buoyancy level the detrainment is effectively greatly increased to represent forced detrainment (Gregory and Rowntree, 1990) as a consequence of vigorous turbulent mixing as the plume is attempting to penetrate the fluid against the buoyancy force. A similar approach is taken here. Initially $\delta$ will be assumed small compared to $\varepsilon$ while the density of the plume is greater than that of the environment. Once the plume passes its zero buoyancy level $\delta$ is increased with increasing difference in density between plume and environment. This is represented by using

$$\delta = \epsilon\left(1 - \frac{\Delta\rho}{\Delta\rho_0}\right)$$

hence $\delta<\varepsilon$ for $\Delta\rho>0$ and $\delta>\varepsilon$ for $\Delta\rho<0$. Here $\Delta\rho=\rho_p(z)-\rho_e(z)$ is the density difference between the plume and the environment at height $z$ and $\Delta\rho_0=\Delta\rho(z_0)$, ie. the density difference between plume and environment at the starting point of the plume, depth $z_0$. When the plume passes its zero buoyancy level it rapidly decelerates as the buoyancy force changes direction. Conservation of vertical momentum causes the plume to spread out. The large value which is implied here for the detrainment counteracts the spreading of the plume by removing the water from the plume into the environment. This has the additional benefit of keeping the model numerically stable.

*Ratio of Circumference to square root of cross sectional-area* $\Lambda$

The total circumference of the ensemble $C$ was previously written as $C=\Lambda a^{1/2}$, where $\alpha$ was the total horizontal cross sectional area of the ensemble. Also for each plume $c=\lambda a_p^{1/2}$ where $c$ and $a_p$ refer to an individual plume in the ensemble. Therefore, if there are $n$ identical plumes in the ensemble

$$C = \sum^{n} c = nc = n\lambda(a_p)^{\frac{1}{2}} = n^{\frac{1}{2}}\lambda(nap)^{\frac{1}{2}}$$

but

$$a = \sum^{n} a_p$$

is the total area and so $C$ can be written as

$$C = \Lambda a^{\frac{1}{2}}$$

where $\Lambda=n^{1/2}\lambda$

$\lambda$ is the ratio of circumference to square root of area for an individual plume and will depend upon the shape of the plumes cross-section. This shape is assumed to be the same for all depths, it only varying in size. For a circle $\lambda\approx3.5$, for a square $\lambda=4$ and for a hexagon $\lambda=3.72$. A value of $\lambda=3.75$ will be used.

*Number of plumes in the ensemble n*

In order to know $n$ both the plume size and the ensemble size must be known. The ensemble size (or Mass flux) has been deduced from the surface boundary conditions. A possibility for the individual plume size is to use linear Rayleigh number theory with eddy diffusion coefficients to deduce the wave lengths for the most rapidly growing instabilities as is done by Jones and Marshall (1992). This remains an area requiring further work.

## Boundary Conditions Parameters

*Critical Rayleigh number $Ra_c$*

Theoretical values for $Ra_c$ using molecular values suggest that it should be around 650 (Turner, 1973). Jones and Marshall (1992) develop the linear theory to take account of eddy diffusion coefficients as well as differing horizontal and vertical coefficients. Their results lead to values which are significantly higher when the horizontal diffusion coefficient is greater that the vertical one. Schott and Leaman calculated a critical Rayleigh number from their Mediterranean data with a value of 7, two orders of magnitude less than that calculated theoretically for molecular scales.

A value of 7 will be used for initial tests of the model as these will be compared with Mediterranean data. Further work will be required later.

*Depth of surface layer $z_0$*

The depth of the layer $z_0$ is the depth of the density inversion. It will depend upon the mixing rates within the surface layer and therefore on the external turbulent and buoyancy forcing. Since the environmental potential temperature and salinity is known a priori $z_0$ can be obtained from them as $z_0=L_{inv}=z(\rho=\rho_{min})$ where $\rho_{min}$ is the minimum potential density in the environmental profile.

The above definition may cause problems during long periods of cooling, for example, when a deep inversion may develop (100s of metres) with a shallower more intense inversion on shorter time scales. The shallower inversion will be more representative of the layer where convection is being initiated since it is within this layer that the turbulent mixing is acting. An appropriate depth scale for this layer is the Moonin-Obukhov length scale. This length scale represents a depth where the externally generated forcing is balanced by the buoyancy forcing. For depths greater than the Moonin-Obukhov depth scale, surface initiated buoyancy forcing dominates over surface driven turbulence mixing. It is therefore reasonable to assume that the plumes start somewhere near the Moonin-Obukhov depth and to take

$$z_0=\min(L_{Mo},L_{inv})$$

where $L_{inv}$ is the inversion depth defined above and $L_{Mo}$ is the Moonin-Obukhov length

scale defined as

$$L_{Mo} = \frac{u_*^3}{kB}$$

where k=0.4 is the von Karman constant, $u_*$ is the friction velocity (obtained from the surface forcing) and B is the surface buoyancy flux

$$B \simeq \frac{g}{\rho_0}\left(-\frac{\alpha H_T}{c_p} + \beta H_S\right)$$

where $\rho_0$ is a reference density, $\alpha$ and $\beta$ are the heat and salt expansion coefficients respectively, $c_p$ is the specific heat capacity for water and $H_T$ and $H_S$ are the surface heat and salt fluxes respectively.

## 5. SUMMARY

There is a clear need for a parametrization scheme for deep convection which has the characteristics of an ensemble of plumes. Plume ensemble equations which describe an ensemble of plumes within an area of the ocean were derived as a first stage to a parametrization scheme. These were initialised by a theory describing a surface layer of the ocean characterised mainly by the external buoyancy and turbulent forcing. Parameters required for the plume model were discussed in turn and suggestions made for suitable values.

The implementation of the plume ensemble model into an Ocean General Circulation Model and how the plumes affect the surrounding environment will be described in a companion paper.

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